32 Simulation of Combustion in a Spark Ignition Engine Using Hybrid Fractal Flame Model

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A new combustion model has been introduced for the turbulent propagating flame in an SI engine. The model is a hybrid model formulated by the concept that the flame front consists of the laminar mixing flamelet and turbulent dissipation flamelet. The fractal dimension is given by the Fractal Dimension Growth Model. The computed results were compared with measured pressures for several different equivalence ratios and engine speeds. It is shown that the hybrid model is the best among other models in predicting pressures for different conditions.

Key Words: **Engine Combustion, Computer Application, Combustion Model, Fractal Flame Model, Turbulence Dissipation Model**

1. IINTRODUCTION

For the prediction of the combustion process in SI engines several different combustion models have been evolved, which were derived from quite different concepts from each other. These models were categorized into the groups of laminar mixing flamelet concept and the turbulent dissipation flamelet concept. Comparing with experimental results, the predictions, however, have not always been satisfactory; it is good in some cases, but not good in other cases. The difficulties are mainly due to the following natures that have not yet been introduced into a combustion model satisfactorily: 1) the burning rate is weak in the early phase of combustion and then grows stronger[1], 2) flame wrinkles become finer as pressure increases[2], 3) the burning rate depends strongly on engine speed.

The purpose of this study is to overcome these difficulties by introducing a new model that combines the laminar mixing and turbulent dissipation concept. The computed results are compared with measured data for several different equivalence ratios and engine speeds.

2. HYBRID FRACTAL FLAME MODEL (HFFM)

The intermittency of turbulence, on which the fractal theory is based, derives the volume ratio, γ_K, of the eddies in Kolmogorov length scale, ℓ_K, to the eddies in the most energetic length scale, ℓ, as follows:

$$\gamma_K = \left(\ell_K / \ell\right)^{3-D_T} \tag{1}$$

where D_T is the fractal dimension expressing the intermittency of turbulence. The turbulent energy dissipation takes place in this volume associated with combustion when burned and unburned eddies meat each other. The combustion can be assumed to be dissipation-controlled, because the reaction rate is usually much higher than the dissipation rate.

It can be naturally assumed that the rest of the volume, $1-\gamma_K$, is filled with turbulent eddies in the scales larger than Kolmogorov scale and with laminar gas blobs which fill spaces between the intermittent turbulent eddies. The eddies larger than Kolmogorov eddy transfer turbulence energy from large scale eddies to small scale eddies while mixing at the interfaces between them due to viscosity except the inertia subrange and producing the laminar wrinkled flame when burned and unburned eddies meet each other.

To summarize, the model are composed of two different kinds of flamelet: the turbulent dissipation flamelet in Kolmogorov scale and the laminar flamelet in scales larger than Kolmogorov scale. The idea is schematically shown in Fig. 1.

The mean burning rate, $\overline{\omega}$, is composed of the burning rate due to the turbulent dissipation, $\overline{\omega}_T$, and that due to laminar mixing, $\overline{\omega}_L$, and is expressed in the form as :

$$\overline{\omega} = \left\{\overline{\omega}_T \gamma_K + \overline{\omega}_L (1-\gamma_K)\right\} f_R \tag{2}$$

$$\overline{\omega}_T = \overline{\rho}_u \frac{1}{\tau_K} \overline{c}(1-\overline{c}) f_W \tag{2a}$$

$$\overline{\omega}_L = \overline{\rho}_u S_t \left|\nabla \overline{c}\right| \tag{2b}$$

where c is progress variable, S_t turbulent burning velocity, τ_K Kolmogorov time scale, ρ_u density of the unburned gas; f_R is a model function for the flame radius effect and f_W denotes a model function for the wall effect, which are described in a later section; the symbol over bar $^-$ indicates ensemble averaged quantity.

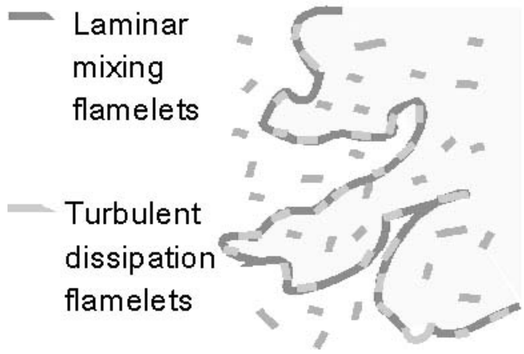

Fig. 1 Concept of Hybrid Fractal Flame Model

The mass of fuel consumption per unit time and volume, $\overline{R}_{fu}$, is given by

$$\overline{R}_{fu} = C_{model}\overline{\omega}(y_{fu,u} - \overline{y}_{fu,b}) \qquad (3)$$

where C_{model} is a model constant, $y_{fu,u}$ is the fuel mass fraction in an unburned mixture and $y_{fu,b}$ the fuel mass fraction in the burned gas. As the relationship between the most energetic length scale, ℓ, and the integral length scale ℓ_I is vague quantitatively, ℓ is assumed ℓ_I for the present study.

2.1 Turbulent Dissipation Flamelet

The formulation of equations for the fractal dimension D_T and Kolmogorov time scale τ_K uses the turbulence energy spectrum function, the relationships between the turbulence variables in a three-dimensional space and in a D_T-dimensional space. The other equation to integrate in the formulation is the empirical coefficient C' of the $k-\varepsilon-\ell$ relation (k : turbulent kinetic energy, ε : dissipation rate of turbulent kinetic energy). The equation for C' is expressed as a function of turbulence Reynolds number R_t as follows:

$$C' = 0.201(1+5.30R_t^{-1/2})^{3/2}, \qquad R_t = \frac{k^2}{\varepsilon\nu} \qquad (4)$$

$$\varepsilon = C'\frac{k^{3/2}}{\ell} \qquad (5)$$

where ν is the kinematic viscosity.

The details of the formulation are shown in Ref. [3] and only the final formulae are described below:

$$D_T = D_T^{*}F_D \qquad (6)$$

$$D_T^{*} = 5 - 1.03\alpha^{*}(1+5.30R_t^{-1/2}), \alpha^{*} = 2.27$$

$$F_D = 1 + \frac{1}{2.61R_t^{0.276} - 5}$$

$$\frac{1}{\tau_K} = \left(\frac{\varepsilon}{\nu}\right)^{1/2}\left(\frac{\ell_K}{\ell}\right)^{(3-D_T)/2} \qquad (7)$$

$$\frac{\ell_K}{\ell} = L^{*}F_L \qquad (8)$$

$$L^{*} = \left\{\frac{6\alpha^{*}}{(1+D_T)}\frac{1}{C'^{4/3}R_t}\right\}^{3/(1+D_T)}$$

$$F_L = \frac{0.199R_t^{0.805} - 2.19}{0.199R_t^{0.805} - 1.19}$$

2.2 Laminar Flamelet

The burning rate due to laminar mixing flamelet is formulated on the basis of the fractal flame concept with the fractal dimension of laminar flame wrinkling, D_L. The fractal dimension D_L is different from D_T, because the laminar wrinkling flame surface is continuous and not intermittent. The ratio of a turbulent flame speed S_t to laminar flame speed S_ℓ is expressed in the following form:

$$\frac{S_t}{S_\ell} = \left(\frac{\varepsilon_i}{\varepsilon_o}\right)^{2-D_L} \qquad (9)$$

where ε_o and ε_i are the outer and inner cutoff scales, respectively. The outer cutoff scale ε_o is taken to be ℓ for the present purpose and the inner cutoff scale ε_i is given by the empirical expression as a function of Karlovitz number presented by Gülder et al[4]. Equation (9) is reduced to the form as:

$$\frac{S_t}{S_\ell} = \left(3.21C_iC'^{1/4}\frac{\ell_K}{\ell}\right)^{2-D_L} \qquad (10)$$

where C_i=2.5.

The laminar flame speed S_ℓ is given by the empirical equation presented by Metghalchi et al.[5].

The fractal dimension D_L for the laminar flamelet is solved by the empirical differential equation, called Fractal Dimension Growth Model, that was developed by Suzuki et. al[1], and described briefly in the following section.

2.3 Fractal Dimension Growth Model(FDGM)

It is a well-known experimental evidence that the flame structure of SI engine combustion grows as the flame propagates during an early phase following spark ignition. To express this nature, several models[6, 7, 8, 9] have been proposed for reducing the burning rate during the early phase of flame propagation. These models were constructed with respect to theoretical or phenomenological considerations, not from measurements of the turbulent flame structure itself. Suzuki et al.[1] measured the fractal dimension of the flame during an early phase of flame propagation in an SI engine, and indicated that the fractal dimension grows as the flame propagates. Upon analyzing the data, they derived an empirical equation for the growth rate of the fractal dimension, which was expressed as a function of turbulence intensity and pressure, as shown below:

$$\frac{dD_2}{dt} = -\frac{1}{\tau}\{D_2 - D_{2\infty}\} \tag{11}$$

$$D_{2\infty} = D_{2T} - (D_{2T} - 1.0)\exp\left[-c_0\left(\frac{\bar{p}}{p_0}\right)^{\gamma}\frac{u'}{S_\ell}\right]$$

$$;\ D_{2T} = 1.32,\quad c_0 = 0.13,\quad \gamma = 0.72$$

$$\tau = \tau_{C0}\left(\frac{\bar{p}}{p_0}\right)^{\phi}\left(\frac{u'}{S_\ell}\right)^{\zeta}$$

$$;\ \tau_{C0} = 5.80\,[\mathrm{ms}],\quad \phi = -0.34,\quad \zeta = -0.93$$

where, D_2 is the fractal dimension of a flame cross section curve; p is pressure, p_0 atmospheric pressure, and u' turbulence intensity. The fractal dimension D_L of a flame surface is obtained from the relationship $D_L = D_2+1$.

When $D_{2\infty}$ and τ are constant, Equation (12) can be integrated to give

$$D_2 = (1.0 - D_{2\infty})\exp\left(-\frac{t}{\tau}\right) + D_{2\infty} \tag{13}$$

Equation (13) shows that D_2 is 1 at time t=0 and then grows toward $D_{2\infty}$ as a period of time elapses.

2.4 Flame Radius Effect f_R

Probability density function of the progress variable, F_c, in the turbulent flame region can be assumed to be the normal distribution in the direction normal to the average flame surface as given below,

$$F_c(r, r_f) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(r-r_f)^2}{2\sigma^2}\right\} \tag{14}$$

where r_f is the radial distance of the average flame front from the spherical flame center, r is the radial distance of the flame front in the turbulent flame region and σ is the standard deviation.

The average flame surface is nearly spherical in shape until it reaches a piston surface. Since the burning rate $\overline{\omega}$ is proportional to the average flame surface area, the distribution of $\overline{\omega}$ may be written as a function of r and r_f as follows:

$$F_\omega(r, r_f) = \left(\frac{r}{r_f}\right)^2\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(r-r_f)^2}{2\sigma^2}\right\} \tag{15}$$

where F_ω is the distribution of $\overline{\omega}$. The value of $(r/r_f)^2$ in the range of the turbulent flame region is especially large in the early phase of the flame development. The flame radius effect on the burning rate distribution is written in a dimensionless form as shown below:

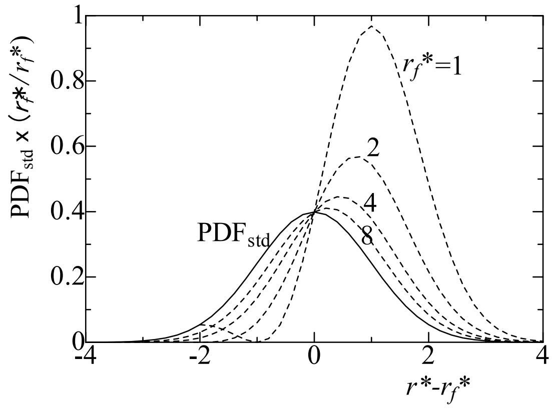

Fig. 2 Flame radius effect on burning rate distribution

$$F_\omega{}^*(r^*, r_f{}^*) = F_\omega(r, r_f)\sigma$$
$$= \left(\frac{r^*}{r_f{}^*}\right)^2\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{(r^*-r_f{}^*)^2}{2}\right\} \tag{16}$$

where, $r^*=r/\sigma$ and $r_f^*=r_f/\sigma$.

Figure 2 shows $F_\omega^*(r^*, r_f^*)$ as a function of $r^* - r_f^*$ in the condition of $r_f{}^* \geq 1$. When r_f^* approaches infinity, $F_\omega^*(r^*, r_f^*)$ reduces to the standard normal distribution. As seen in the figure, the average burning rate distribution (maximum point) increases with decreasing r_f^* or it is larger at the outer location from the average flame front. The maximum value of $F_\omega^*(r^*, r_f^*)$ is given by,

$$(F_\omega{}^*)_{\max} = F_\omega{}^*((r^*-r_f{}^*)_{\max}, r_f{}^*) \tag{17}$$
$$(r^*-r_f{}^*)_{\max} = \frac{1}{2}\left(-r_f{}^* + \sqrt{r_f{}^{*2}+8}\right)$$

Then the function f_R in Eq. (1) is

$$f_R = \frac{F_\omega{}^*((r^*-r_f{}^*)_{\max}, r_f{}^*)}{F_\omega{}^*(0,\infty)}$$
$$= \frac{F_\omega{}^*((r^*-r_f{}^*)_{\max}, r_f{}^*)}{0.3989} \tag{18}$$

The standard deviation σ is assumed to be the integral length scale.

2.5 Wall Effect f_W

The function f_W in Eq. (1) is to take into account a decrease in probability of burned and unburned gas blobs meeting each other in the turbulent dissipation flamelet regime in near wall regions. The wall effect function f_W is expressed in the form as[10]

$$f_w = \left(\frac{\ell_I}{C_\ell \ell_T}\right)^2 \quad , \quad \frac{1}{C_\ell^{\,2}} \le f_w \le 1 \qquad (19)$$

$$\frac{\ell_I}{\ell_T} = \left(\frac{C_\mu}{15}\right)^{1/2} \left(\frac{2}{3}\right)^{1/4} R_t^{\,1/2}$$

where ℓ_I is the integral length scale, ℓ_T the Taylor micro scale, $C_\mu = 0.09$, and C_ℓ is a model constant, which is set to 2.0 in the present study.

2.6 Outline of Governing Equations

Governing equations to be solved are the general conservation equations in the Reynolds averaged forms for mass, momentum, average enthalpy (over unburned and burned gases), enthalpy of unburned gas[3], progress variable, the turbulence kinetic energy and its dissipation rate. The standard *k*-*ε* model was employed for the turbulence model. The wall boundary conditions for the momentum and the enthalpies were given by the law of the wall.

3. COMPUTATIONAL RESULTS AND COMPARISON WITH MEASUREMENTS

3.1 Test Engine and Computational Conditions

Two four-stroke single cylinder engines were used for the computations and experiments. Each of them had a disk-shaped combustion chamber and a spark plug was located at the center of the cylinder head. They are named Engine A and Engine B in the text, and their specifications and test conditions are shown in Table 1. Engine A was used for the measurements varying engine speed and Engine B for the data varying the equivalent ratio.

The computations were performed solving a set of the governing equations in an axisymmetric field. The residual gas mass fractions were set to 0.13 and 0.1 for Engine A and Engine B, respectively, estimated by the one-zone global model. The pressure and temperature at the inlet valve closing were assumed for each test condition based on the experiments. The wall temperatures were set to 433 K for both engines given by measurements. The initial value of turbulence kinetic energy was assumed to be 0.74 C_m^2 (C_m: mean piston speed).

3.2 Pressure and Apparent Heat Release Rate;
(a) Engine A

For comparison purpose, two other models of laminar flamelet type also were tested: Fractal Dimension Growth model(FDGM) and Weller model[11]. The former is expressed by setting γ_T=0 and f_R =1. The latter is one of popular models and expressed by the same form as FDGM except that S_t / S_ℓ is given by an empirical relation as a function of u' / S_ℓ .

Table 1 Engine specifications and test conditions

[Engine A]		
Bore x Stroke	: 86.0 x 84.0 mm	
Comp. Ratio	: 7.9	
Fuel	: Regular gasoline	
Engine speed	Equivalence ratio	Ign. Time
1000 rpm	0.954	13 °btdc
1500	1.004	14
1800	1.045	20
[Engine B]		
Bore x Stroke	: 82.6 x 114.3 mm	
Comp. Ratio	: 7.0	
Fuel	: Iso-octane	
Engine speed	Equivalence ratio	Ign. Time
750 rpm	1.0	15 °btdc
750	0.91	16
750	0.83	19

Figure 3 shows the computed pressure and the apparent heat release rate for Engine A at three different engine speeds, compared with the experimental results. The apparent heat release rate was calculated from the pressure data assuming the ratio of specific heats to be 1.4 for both the computation and measurement. The model constant C_{model} in Eq. (2) of each model was adjusted so as to obtain the best fit to the measured pressure for 1000 rpm. The model constants adjusted were 2.48, 2.55 and 2.55 for HFFM, FDGM and Weller model, respectively. Then the computations for 1500 rpm and 1800 rpm were performed using the same value of C_{model} for each model. It is seen that HFFM gives much better results than the other two models, which are very slow in combustion at 1500 and 1800 rpm.

Figure 4 indicates γ_K and the ratio of the turbulent dissipation heat release rate to the total heat release rate $R_{\omega T}$ as a function of crank angle. It is seen that γ_K is only 13% to 17%, being nearly independent of engine speed. In contrast, the turbulent dissipation heat release rate occupies a larger part as the engine speed increases from around 20% at 1000 rpm to 50% at 1800 rpm. The result implies that the Kolmogorov time scale τ_K becomes smaller as the turbulence increases. It may be said that at 1000 rpm the role of the turbulent dissipation heat release rate is not considerable, while at 1800 rpm it is as large as half the total heat release rate.

(b) Engine B

Figure 5 compares the computed pressure and heat release rate with the measurement for three different equivalent ratios at 750 rpm. The models taken for comparison are the HFFM, FDGM and FDGM without the pressure factor on the fractal dimension growth rate[12].

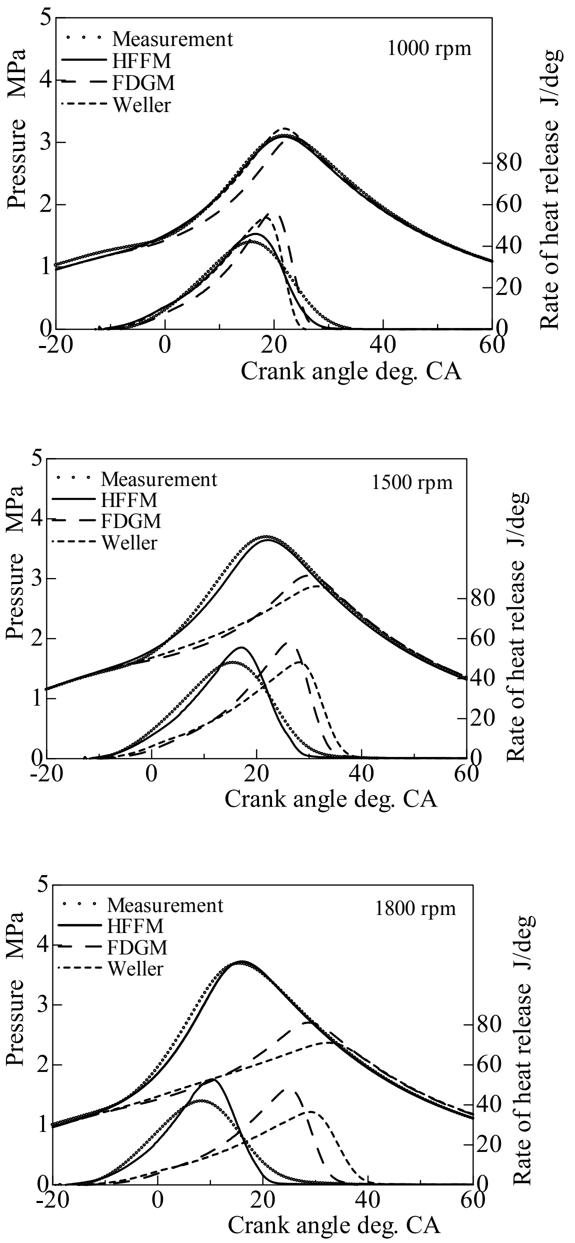

Fig. 3 Comparison of computational results by three different models with measurements for 1000, 1500 and 1800 rpm; Engine A, C_{model}=2.75

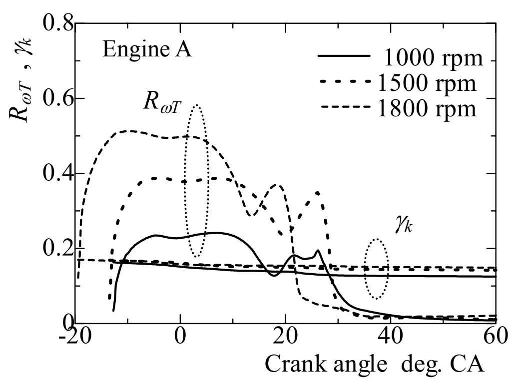

Fig. 4 Contribution of Turbulent dissipation flamelet;
$R_{\omega T} = \omega_T\gamma_k /[\omega_T\gamma_K + \omega_L (1-\gamma_K)]$; Engine A

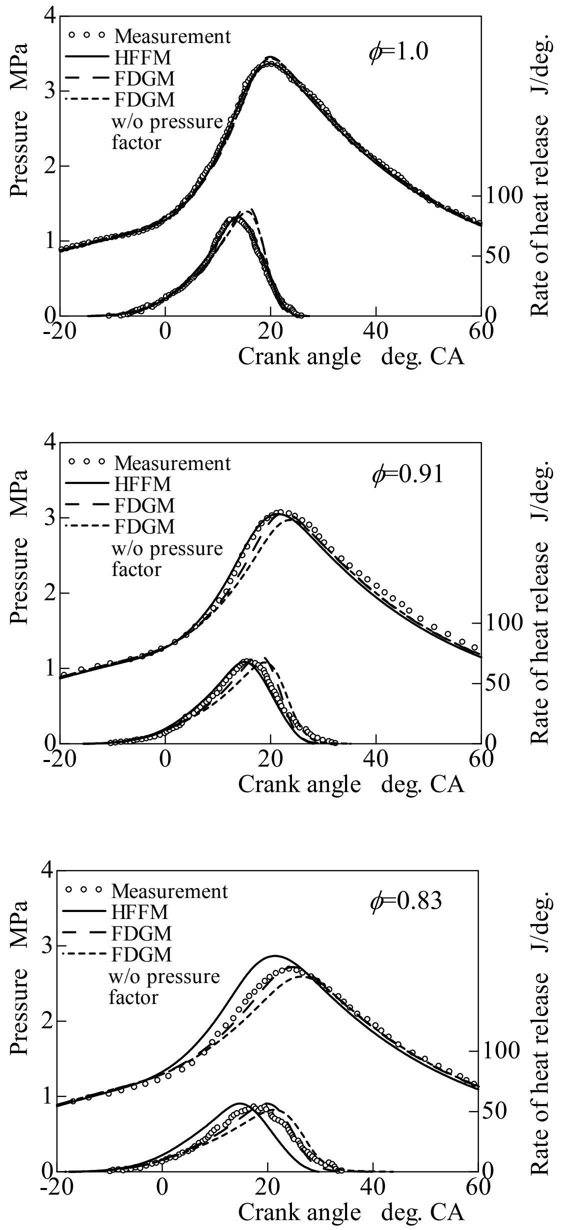

Fig. 5 Comparison of computational results by three different models with measurements for equivalence ratio $\phi = 1.0$, 0.91 and 0.83; Engine B, 750 rpm, C_{model}=1.65.

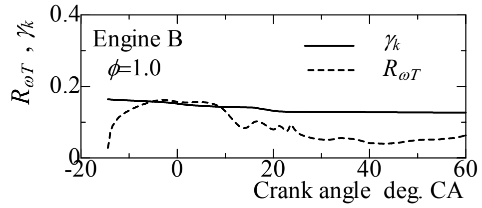

Fig. 6 Contribution of Turbulent dissipation flamelet;
$R_{\omega T} = \omega_T\gamma_k /[\omega_T\gamma_K + \omega_L (1-\gamma_K)]$; Engine B

It is seen that for ϕ=0.91 the HFFM and FDGM coincide well with the measurement while the FDGM without the pressure factor underestimates. In the case of ϕ=0.83, the FDGM gives the best result, the HFFM overestimates and the FDGM without the pressure factor under estimates.

Figure 6 shows the contribution of the turbulent dissipation flamelet for Engine B at ϕ=1.0. The value of γ_k is nearly the same as that for Engine A, while $R_{\omega T}$ is less than 16%, even less than that for Engine A at 1000 rpm.

A general tendency seen in Figs. 4 and 6 is that the flame relies on turbulent-dissipation more as engine speed increases, and that it approaches the laminar flamelet as engine speed decreases.

4. CONCLUSION

A new combustion model has been introduced for the turbulent propagating flame in an SI engine. The model is a hybrid model formulated by the concept that the flame front consists of the laminar mixing flamelet and turbulent dissipation flamelet. For the laminar mixing flamelet, the fractal model was employed and the fractal dimension was given by the Fractal Dimension Growth Model The computed results were compared with measured pressures for several different equivalence ratios and engine speeds. From the comparison of the computed results with the measurements, and the analyses of the results, the following conclusions are drawn from this study.

(1) As a whole, the hybrid model is the best among other models tested in this study in predicting pressures for different conditions.
(2) The laminar flamelet models tend to delay in combustion as engine speed increases.
(3) The analysis of the computed results shows that the flame relies on turbulent dissipation more as the engine speed increases, and that it approaches the laminar mixing flamelet as the engine speed decreases.

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REFERENCES

[1] Suzuki, K. and Nishiwaki, K., "Fractal Dimension Growth Model for SI Engine Combustion," SAE 2004-01-1993, 2004.
[2] Kobayashi, H., Kawahata, T., Seyama, K., Fujimori, T. and Jong-Soo Kim, "Relationship between the smallest Scale of Flame Wrinkles and Turbulence Characteristics of High-Pressure, High-Temperature Turbulent Premixed Flames," Proc. the Combustion Institute, Volume 29, pp. 1793-1800, 2002.
[3] Nishiwaki, K., "Modeling Engine Heat Transfer and Flame-Wall Interaction," Proc. COMODIA 98, pp. 35-44, 1998.
[4] Gülder, O. L. and Smallwood, G. J., Combustion and Flame, Vol. 103, pp.107-114, 1995.
[5] Metghalchi, M. and Keck, J. C. (1982), Combustion and Flame, Vol 48, p.191.
[6] 1.Weller, H. G., Uslu, S., Gosman, A.D., Maly, R. R., Herweg, R. and Heel, B., "Prediction of Combustion in Homogeneous-Charge Spark-Ignition Engines," Proc. COMODIA94, pp. 163-169, 1994.
[7] Zhao, X., Matthews, R. D. and Ellzey, J. D, "Numerical Simulation of Combustion in SI Engines: Comparison of the Fractal Flame Model to the Coherent Flame Model," Proc. COMODIA 94, pp. 157-162, 1994.
[8] Herweg, R. and Maly, R. R.," A Fundamental Model for Flame Kernel Formation in S. I. Engines," SAE Paper 922243, 1992.
[9] Santavicca, D. A., Liou, D. and North, G. L., "A Fractal Model of Turbulent Flame Kernel Growth," SAE Paper 900024, 1990.
[10] Kojima, K. and Nishiwaki, K., "Modeling of flame-wall interaction for combustion and heat transfer in S.I. engines," JSAE Review Vi. 18, pp. 11-17, 1997.
[11] Weller, H. G., Uslu, S., Gosman, A.D., Maly, R.R., Herweg, R. and Heel, B., "Prediction of Combustion in Homogeneous-Charge Spark-Ignition Engines," Proc. COMODIA94, pp. 163-169, 1994.
[12] Suzuki, K. and Nishiwaki, K., "Fractal Dimension Growth in Flame Front Wrinkles during the Early Phase of Flame Propagation in an SI Engine," SAE 2003-01-1840, 2003.